MA 571 Qualifying Exam. August 2018. Professor McClure.

Each problem is worth 14 points and you get two points for free.

Please be careful that your handwriting is clear and easy to read.

Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.

When you use a set theoretic fact that isn't stated in Munkres and isn't obvious, be careful to give a clear explanation.

1. Let $f : X \to Y$ and $g : Y \to X$ be continuous functions with $g \circ f$ equal to the identity map of X. **Prove** that f is a homeomorphism from X to $f(X)$ (with the subspace topology).

2. Suppose that X has a component which is not compact. **Prove** that X is not compact.

3. Let $q : X \to Y$ be a continuous surjective map of topological spaces. Show that if X is compact and Y is Hausdorff then q is a quotient map.

4. Suppose that X is locally compact Hausdorff and that X has exactly two components. Suppose also that neither component is compact. **Prove** that the one-point compactification of X is connected. (You can use the fact stated in Problem **??**, even if you didn't do that problem.)

5. In $\mathbb{R}^3$, let $P = (1, 0, 0)$ and $Q = (-1, 0, 0)$. **Prove** that $\mathbb{R}^3 - \{P, Q\}$ is simply connected. If you use deformation retractions, you don't have to give formulas for the retractions or the homotopies.

6. Let X be the quotient space obtained from an octagon P by pasting its edges together according to the labelling scheme $ada^{-1}bc^{-1}db^{-1}c$

   i) Calculate $H_1(X)$. (You may use anything proved in Munkres, but be careful to be clear about what you're using.)

   ii) Assuming X is homeomorphic to one of the standard surfaces in the classification theorem, which surface is it?

7. Let X, A and B be topological spaces. Let p (resp. q) be the projection $X \times A \to X$ (resp. $X \times B \to X$). Let $f : X \times A \to X \times B$ be a continuous bijection with the property that $q \circ f = p$. Suppose that X is connected and that A and B are discrete. **Prove** that f is a homeomorphism.