Math 571 Qualifying Exam **August 2017**

Each problem is worth ten points, for a total of sixty possible points.

1. Let $\mathbb{R}$ denote the real line (equipped with its usual topology) and let $X = \prod_{n=0}^{\infty} \mathbb{R}$ be the countable infinite product of copies of $\mathbb{R}$, equipped with the box topology (recall that a basis for this topology is given by the collection of subsets of the form $\prod U_n$, for all sequences of open sets $U_n \subset \mathbb{R}$). Show that the diagonal function $\delta : \mathbb{R} \to X$, which sends $t \in \mathbb{R}$ to the constant sequence $\delta(t) = (t, t, t, \ldots) \in X$, is not continuous.

2. Let X be a locally compact topological space and let X^+ denote its one-point compactification. Show that X^+ is compact. Is X^+ necessarily connected?

3. Let $q : Y \to X$ be a continuous surjective map of topological spaces. Show that if Y is compact and X is Hausdorff, then q is a quotient map.

4. Let X be a path connected topological space, let $I = [0, 1]$ denote the unit interval, equipped with its standard topology, and suppose given two continuous functions $f : I \to X$ and $g : I \to X$. Show that f and g are homotopic.

5. Let X be a simply connected topological space and let $p : Y \to X$ be a covering space such that Y is path connected. Show that p is one-to-one and onto.

6. Let n be a positive integer and let $s_1, \ldots, s_n \in S^2$ be a sequence of n distinct points on the 2-sphere S^2. Let $X = S^2 - \{s_1, \ldots, s_n\}$ be the subspace of S^2 obtained as the complement of $\{s_1, \ldots, s_n\} \subset S^2$. Calculate $\pi_1(X, x)$ for a choice of basepoint $x \in X$.