MA 571 Qualifying Exam. August 2009. Professor McClure.

Each problem is worth 14 points and you get two points for free.

Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.

When you use a set theoretic fact that isn't obvious, be careful to give a clear explanation.

1. Let X be a topological space, let A be a subset of X, and let U be an open subset of X. **Prove** that $U \cap \bar{A} \subset \overline{U \cap A}$.

2. Let X be a topological space and let A be a subset of X. Suppose that $\bar{A} = X$ and that A is connected. **Prove** that X is connected.

   Note: this is a special case of a theorem in Munkres. You cannot use that theorem in your proof.

3. Let X and Y be compact topological spaces. **Prove** that $X \times Y$ (with the product topology) is compact.

   Note: Munkres uses a lemma to prove this, and you may also use this lemma.

4. Let X be a topological space and let $\sim$ be the equivalence relation on $X \times [0,1]$ defined by

   $$(x,t) \sim (x',t') \Leftrightarrow (x,t) = (x',t') \text{ or } t = t' = 0$$

   (that is, the only nontrivial equivalence class is $X \times \{0\}$). Let Y be the quotient space $(X \times [0,1])/\sim$.

   Suppose that X is Hausdorff.

   Prove that Y is Hausdorff.

5. A topological space is called *normal* if for each pair C, U where C is closed, U is open, and $C \subset U$ there is a W with W open, $C \subset W$ and $\bar{W} \subset U$.

   Suppose that X is normal and that U, V are open sets with $U \cup V = X$. **Prove** that there are open sets U_1, V_1 with $\bar{U}_1 \subset U$, $\bar{V}_1 \subset V$, and $U_1 \cup V_1 = X$.

6. Let X be a topological space which is homotopy equivalent to a point. Let Y be a path-connected topological space. **Prove** that any two continuous functions $f, g : X \to Y$ are homotopic.

7. Let $p : E \to B$ be a covering map.

   Let Y be locally path-connected.

   Let $g : Y \to E$ be a function (which we do not assume is continuous) such that

- $p \circ g$ is continuous, and
- $g \circ \gamma$ is continuous for every path γ in Y.

Prove that g is continuous.