QUALIFYING EXAMINATION

JANUARY 2002

MATH 571 - Prof. Smith

I. Let $f : E \to B$ be a covering of a compact Hausdorff space B. Prove that E is compact if and only if for each $b \in B$ the fiber $f^{-1}b$ is finite.

II. Let $X = \hom(I, \mathbb{R})$ be the set of all real valued continuous functions on the unit interval $I = [0, 1]$. Prove that the compact open topology on X is second countable, i.e. that it has a countable base.

III. A T_1 space X is completely regular if for every point $x \in X$ and closed subspace $F \subset X$ there is a a continuous real valued function $f \colon X \to \mathbb{R}$ such that $f(x) = 0$ and $f(F) = 1$. Show that every subspace of a compact Hausdorff space is completely regular.

IV. Prove that every locally compact Hausdorff space is completely regular.

V. Let X be a topological space. Let A be a subspace of X, F be a closed subspace of X and Q be a quotient space of X.

A) If X is compact which of A, F and Q must also be compact?

B) If X is connected which of A, F and Q must also be connected?

C) If X is path connected which of A, F and Q must also be path connected?

D) If X is Hausdorff which of A, F and Q must also be Hausdorff?

E) If X is normal which of A and F must also be normal.