**QUALIFYING EXAMINATION**
JANUARY 2001
MATH 571 - Prof. Smith

I. a) Give the definition of a normal topology.
b) Let $M$ be a metric space. Show that the metric topology on $M$ is normal.

II. Let $X$ be a topological space and let $A$ be a subspace of $X$. Answer true or false for each of the following and give a counterexample for those assertions that are false. (You need not prove those assertions that are true.)
a) If $X$ is connected then $A$ is connected.
b) If $X$ is compact then $A$ is compact.
c) If $X$ is Hausdorff then $A$ is Hausdorff.
d) If $X$ is compact and $A$ is a closed subset of $X$ then $A$ is compact.
e) If $X$ is metrizable then $A$ is metrizable.

III. Let $X$ be a Hausdorff topological space that is the image of a continuous function $f : [0, 1] \to X$. Show that
a) $X$ is connected,
b) $X$ is compact,
c) $X$ is normal,
d) $X$ has a countable dense subset.

IV. Let $R^n$ denote $n$-dimensional space, so that $R$ is the real line and $R^2$ is the plane.
a) Show that $R$ is not homeomorphic to $R^2$.
b) Show that $R^2$ is not homeomorphic to $R^3$.

V. Let $p : E \to B$ be a covering map. If $B$ is compact and $p^{-1}(b)$ is finite for each $b \in B$, then $E$ is compact.