QUALIFYING EXAMINATION
AUGUST 2001
MATH 571 - PROF. J. SMITH

1.(10 pts) Let X be a compact space and let

$$A_1 \supset A_2 \supset \cdots \supset A_k \cdots$$

be a descending chain of non-empty closed subsets of X. Show that the intersection $\cap_{k=1}^{\infty} A_k$ is not empty.

2.(10 pts) Show that topology of a compact metric space has a countable basis.

3.(10 pts) Show that the product of connected spaces is connected.

4.(10 pts) If X is a simply connected space (i.e. every map $S^1 \to X$ is homotopic to a constant map) then every map $X \to S^1$ is homotopic to a constant map. Hint: Use the universal cover of S^1.

5.(20 pts) In each of the following describe a homeomorphism between the two topological spaces or show there is none.

- $\mathbb{R}$ and $\mathbb{R}^2$
- $\mathbb{R}^2$ and $D^2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1$
- $[0, 1) \times (0, 1)$ and $[0, 1) \times [0, 1]$
- The parabola $y = x^2$ with the subspace topology and $\mathbb{R}$

6.(10 pts) Let M be a compact metric space, X be a Hausdorff space and $f\colon M \to X$ be a continuous surjection. Show that X is metrizable.