Qualifying Examination

January 1999

MATH 571 - Prof. Gottlieb

Each problem is worth **20 points**.

1. Let $\mathbb{Q}$ be the rational numbers with the order topology.
   a) Show that $\mathbb{Q}$ is not discrete.
   b) Show that every connected component of $\mathbb{Q}$ is a point.

2. Let $f : X \to Y$ be a continuous map and let $A \subset X$ be a subset.
   a) Is $f(\overline{A}) \subset \overline{f(A)}$? Give a proof or counterexample.
   b) Is $\overline{f(A)} \subset f(\overline{A})$? Give a proof or counterexample.

3. a) Define the notion of homotopy equivalence of spaces.
   b) Show that homotopy equivalence is an equivalence relation on spaces.

4. Show the countable product I^w is a compact metric space by defining a metric. Here I is $[0, 1]$.

5. A *proper map* is a continuous map $f : X \to Y$ so that f^{-1} (compact set) is compact in X. Assume X and Y are Hausdorff.
   a) Show f is proper if f is closed and every fibre $f^{-1}(y)$ is compact.
   b) Give an example of a map which has all its fibres compact and yet is not proper.
   c) What condition on $\pi_1(X)$ is necessary and sufficient so that the universal covering space map $p : \tilde{X} \to X$ is proper?