QUALIFYING EXAMINATION
MATH 571
JANUARY 1994

1. Let $f, g : X \to Y$ be continuous. Define $h : X \to Y \times Y$ by $h(x) = (f(x), g(x))$. Show that if f is an embedding then h is an embedding.

2. Let $f : X \to Y$ be continuous. Let C be a connected component of Y. Show that $f^{-1}(C)$ is a union of connected components of X.

3. Let $f : X \to Y$ be continuous, closed, and surjective. Show that if X is locally connected then Y is locally connected.

4. Let X be locally compact Hausdorff and let X^∞ be its one point compactification. Let Y be a compact Hausdorff space such that $X \subset Y$ and X is open and dense in Y.

   (a) Show that there is a continuous function $f : Y \to X^\infty$ such that $f(x) = x$, $x \in X$.

   (b) Show that such an f is unique.

5. Let $p : E \to B$ be a covering space with E path connected and locally path connected. Let $f : E \to E$ be continuous such that $pf = p$. Show that f is a homeomorphism.

6. Let X be locally compact Hausdorff and Y a space. Let $C(X, Y)$ be the space of continuous functions from X to Y with the compact-open topology. Let Z be a space. Given $f : X \times Z \to Y$ define $\hat{f} : Z \to C(X, Y)$ by $\hat{f}(z)(x) = f(x, z)$. Show that f is continuous if and only if $\hat{f}$ is continuous. (You may use the fact that the evaluation map $e : X \times C(X, Y) \to Y$ is continuous.)

7. Let $p : R \to S^1$ be the exponential map. Define $\psi : \pi_1(S^1; s_0) \to Z$ by $\psi([\sigma]) = \tilde{\sigma}(1)$, where $p\tilde{\sigma} = \sigma$ and $\tilde{\sigma}(0) = 0$. (Here $s_0 = (1, 0)$.)

   (a) Show that ψ is well defined.

   (b) Show that ψ is injective.