NAME: ______________________________

T.T.Moh **Math 554 Qualifying Examination** **Jan 8 2015**

- This is a two hour test.
- Write your answers on the test paper!
- For decimal approximations, it is enough to give 2 decimal places.
- Show your work such that your reasoning can be followed.
- There are 10 pages, 10 questions, 20 points each and 200 points on this test.

1. A ring is said to be *noetherian* if every ideal is finitely generated. Show that a ring is noetherian iff any submodule of any finitely generated module is finitely generated.

2. Show that all integer valued polynomials at integers (i.e., $f(n) \in Z$ for all $n \in Z$) in $Q[x]$ form a vector space and find a basis for it.

3. Let A be the following matrix,

$$A = \begin{bmatrix} 1, & 2, & 3 \\ 4, & 5, & 6 \\ 7, & 8, & 9 \end{bmatrix}$$

Let R^3 , the real space, be considered as a $R[x]$ module defined by

$$f(x) \cdot v = f(A) \cdot v$$

What are the torsion factors and the betti numbers?

4. Compute the elementary factors to decide if the following two matrices are similar.

$$\begin{bmatrix} 2, & -2, & 14 \\ 0, & 3, & -7 \\ 0, & 0, & 2 \end{bmatrix}, \qquad \begin{bmatrix} 2, & 2, & 1 \\ 0, & 2, & -1 \\ 0, & 0, & 3, \end{bmatrix}$$

5. Find the *Jördan* canonical form of the following matrix

$$A = \begin{bmatrix} 1, & 1, & 1 \\ 0, & 0, & 0 \\ 0, & 0, & 0 \end{bmatrix}$$

6. Show that a square idempotent matrix A (i.e., $A^2 = A$) can be diagonalized.

7. (Reflections) Let us consider the reflections of R^3. A reflection is a 3×3 matrix R such that (1) det(R)=-1. (2) it fixes the length of vectors. Show that -1 must be an eigenvalue of a reflection.

8. Check the Cayley-Hamilton Theorem of the following matrix A by computation,

$$A = \begin{bmatrix} 1, & 2, & 0 \\ 2, & 1, & 1 \\ 0, & 1, & 0 \end{bmatrix}$$

9. Let V be a complete inner product space. Show that $V \neq U \oplus U^{\perp}$ if U is not closed.

10. Decide if the following function has a local minima at the origin;

$$f(x, y, z) = 3x^2 + 4xy + 2xz + 2y^2 + 3yz + 2z^2$$