QUALIFYING EXAMINATION
JANUARY, 2008
MATH 554

You have to show your work.

1. Let A be the following matrix,

$$\begin{pmatrix} 0 & 1 & 0 \\ 6 & 11 & 12 \\ -4 & -7 & -8 \end{pmatrix}$$

(5 pts) (a): Find all eigenvalues of A.

(5 pts) (b): Is A diagonalizable?

(15 pts) **2.** A reflection A on $\mathbb{R}^n$ is defined to be a linear transformation A of $\mathbb{R}^n$ which preserves the length of all vectors and reverse the orientation (i.e., with determinant $(A) = -1$). Let A be a reflection on $\mathbb{R}^3$, show that -1 is an eigenvalue of A.

(15 pts) **3.** Express the commutative group $\mathbb{Z}^3/(f_1, f_2, f_3)$ where $f_1 = (1, 2, 3)$, $f_2 = (4, 6, 8)$, $f_3 = (6, 10, 12)$ as a direct sum of cyclic groups.

(10 pts) **4.** A matrix A is said to be *idempotent* if $A^m = I$ for some $m \geq 1$. Show that a symmetric real matrix is idempotent iff $A^2 = 1$.

(10 pts) **5.** Let $<,>$ be an inner product of a finite dimensional complex vector space V, and A a self-adjoint operator of V, Show that $< Av, v >$ is always a real number for any $v \in V$.

(10 pts) **6.** Find the area of the convex pentagon in $\mathbb{R}^2$ with vertices $(0,0), (6,0), (8,3), (5,6), (0,4)$.

(10 pts) **7.** Let $\mathbb{P}_3$ be the vector space of all real polynomials of degree 3 or less. Let the inner product $(f|g)$ be defined as $\int_0^1 fg\ dx$. Find an orthonormal basis of $\mathbb{P}_3$.

8.

(10 pts) (a): Find the Jordan canonical form J of the following matrix over complex numbers

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(10 pts) (b): Find a matrix M such that $J = M^{-1}AM$.