QUALIFYING EXAMINATION
JANUARY 2004
MATH 553 - Prof. Moh

1) (10 points) Find the prime decomposition of $7+4i$ in the Gaussian intergers $\mathbb{Z}$[i].

2) (10 points) Let the *complex projective line* $\mathbb{P}^1_{\mathbb{C}} = \mathbb{C} \cup \infty$, here $\mathbb{C}$ is the set of complex numbers. Let **G** be the *fractional linear transformation group* defined as follows,

$$G = \{\pi : \pi(x) = \frac{ax+b}{cx+d}, ad - bc \neq 0\}$$

Let **T**=$\{0, 1, \infty\}$. Find Stab(**T**).

3) (10 points) Find the center of the group of all *rigid motions*, i.e., all mappings which preserve distances, of $\mathbb{R}^2$ where $\mathbb{R}$ is the set of all real numbers.

4) (10 points) Suppose that $o(\mathbf{G}) < 60$ where $\mathbf{G}$ is a simple group. Show that $o(\mathbf{G})$ is a prime number.

5) (10 points) Find all composition series of the dihedral group $\mathbf{D}_4$.

6) (10 points) Prove that for any $n \geq 1$ the following polynomial in $\mathbb{Q}[x]$ has no multiple root,

$$f(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$$

7) (10 points) Show that $\mathbb{Z}[i]/(3+2i)$ is a field where $\mathbb{Z}[i]$ is the ring of Gaussian integers. What is its characteristic?

8) (10 points) Let α satisfies an irreducible polynomial of odd degree in $\mathbb{K}[x]$. Show that $\mathbb{K}[\alpha] = \mathbb{K}[\alpha^2]$.

9) (10 points) Show that $[\mathbb{Q}[e^{\frac{2\pi i}{8}}] : \mathbb{Q}]$=2.

10) (10 points) Let $\mathbb{L}$ be the splitting field of the polynomial $x^4 - 2$ over $\mathbb{Q}$. Find the Galois group of $\mathbb{L}$ over $\mathbb{Q}$.