**QUALIFYING EXAMINATION**

AUGUST 2003

MATH 544 - Prof. Davis

(15 pts) 1. Let $f$ be an integer valued function on $\mathbb{R}$. Show that $\{x : f \text{ is not continuous at } x\}$ is a Borel set.

(15 pts) 2. Let $A$ and $B$ be (not necessarily Lebesgue measurable) subsets of $\mathbb{R}$ and let $|\ |_e$ stand for Lebesgue outer measure. Prove that if $|A|_e = 1$ and $|B|_e = 1$ and $|A \cup B|_e = 2$ then $|A \cap B|_e = 0$.

(15 pts) 3. Show, with proof, $F'(1) < 0$, where $F(x) = \displaystyle\int_0^\infty \frac{e^{-xy}}{y^3+1}\,dy$.

(15 pts) 4. Prove that if $f$ is a uniformly continuous function on $\mathbb{R}$ and if $h(x) = \displaystyle\int_{x-1}^{x+1} f(s)ds$ then $h$ is uniformly continuous on $\mathbb{R}$.

(20 pts) 5. Let $g(x)$ be a continuous function on $[0,1]$ satisfying $g(0) = 0$, $g(1) \le 1$, and $g(s) \le g(t)$ if $0 \le s < t \le 1$. Put $\phi_n(x) = g(x)^n$. Prove that if $f$ is a continuous function on $[0,1]$, $\lim_{n\to\infty} \displaystyle\int_0^1 f(x)d\phi_n(x)$ exists.

(20 pts) 6. Let $f$ be a bounded Lebesgue measurable function on $\mathbb{R}$. Put
$g(x) = \sup\{a \in \mathbb{R} : |\{y : y \in (x, x+1) \text{ and } f(y) > a\}| > 0\}$,
where $\|$ is Lebesgue measure (i.e. $g(x)$ equals the essential supremum of $f$ over $(x, x+1)$). Prove $\liminf_{x\to 0} g(x) \ge g(0)$.