

QUALIFYING EXAMINATION

AUGUST 2005

MATH 530 - Prof. Drasin

Each problem is worth 5 points.

As in all math exams, you have to fully justify your answers.

1. How many zeros does the polynomial $z^6 - 5z + 20$ have in the annulus $1 < |z| < 2$?

2.
$$\int_{-\infty}^{\infty} \frac{dx}{x^3 - i} = ?$$

3. Given $N = 0, 1, 2, \dots$, find all entire functions f such that $|f(z)| \leq |z|^N$ for all $z \in \mathbb{C}$.

4. Let γ be the ellipse $9x^2 + 16y^2 = 144$. [i.e. $9(\operatorname{Re} z)^2 + 16(\operatorname{Im} z)^2 = 144$], and

$$\varphi(z) = \int_{\gamma} \frac{e^{2|\zeta|}}{\zeta - z} d\zeta.$$

- (a) For which R can we be sure that φ is analytic in the disc $\{z : |z| < R\}$?

- (b) Estimate $|\varphi''(0)|$ from above.

5. Find a conformal map φ from the strip $\{x+iy : |x-y| < 1\}$ onto the first quadrant. You may express φ as a finite composition of simpler explicit mappings.

6. Is there a holomorphic function h in the disc $\{z : |z| < 1\}$ that satisfies

$$\lim_{|z| \rightarrow 1} |h(z)| = \infty?$$