QUALIFYING EXAMINATION

JANUARY 2003

MATH 530 - Prof. Bell

1. *(10 pts)* Suppose P and Q are polynomials and that the degree of P is at least two less than the degree of Q. Prove that the sum of all the residues of P/Q in the complex plane is zero.

2. *(20 pts)*

a) Prove that an uncountable subset of a domain in the complex plane must have a limit point in the domain.

b) Prove that an analytic function on a domain can have at most countably many zeroes if it is not identically zero.

c) Does there exist a non-constant analytic function on the unit disc with infinitely many zeroes on the unit disc? Prove or give a counterexample.

3. *(20 pts)* Prove that if a harmonic function on a domain vanishes on an open subset of the domain, then it is identically zero. Can it vanish on a set with a limit point in the domain and not be identically zero?

4. *(20 pts)* Suppose that $f(z)$ has an essential singularity at $z = a$. Prove that there is a sequence of points z_n tending to a such that $(z_n - a)^n f(z_n)$ tends to ∞.

5. *(10 pts)* Does there exist a sequence of polynomials $P_n(z)$ which converges uniformly to $1/z$ on $\{z : |z| = 1\}$? Explain.

6. *(20 pts)* Suppose that f is a continuous complex valued function on a domain Ω. Prove that if f^2 and f^3 are analytic on Ω, then f itself must be analytic.