

**QUALIFYING EXAMINATION**  
AUGUST 2000  
MATH 530 - Prof. Drasin

(15 pts) 1. Find a function which is meromorphic in  $\hat{\mathbb{C}}$  with a pole of order 3 at  $z = 0$ , a removable singularity at  $z = 2i$ , a simple zero at  $z = -2i$  and a zero of order two at  $\infty$ .

(20 pts) 2. Consider the “function”  $\sqrt{z(z-1)}$ . In what annuli  $\{a < |z| < b\}$  can we find a single-valued branch of this expression? How many such branches are there? Indicate the Laurent series in the appropriate annulus/annuli.

(15 pts) 3. Let

$$h(z) = \int_{-1}^1 e^{-zt} \sin |z| dt.$$

Prove that  $h$  is an entire function and compute  $h!$  Be sure to justify all steps.

(15 pts) 4. In what region is

$$F(z) = \sum_1^{\infty} e^{z \ln n}$$

analytic? Be sure to give a proof.

(20 pts) 5. For what real  $p$  does  $\int_0^{\infty} \frac{x^p}{1+x} dx$  converge? Compute this integral for all such  $p$ .

(15 pts) 6. Map the region bounded by  $|z| < 1$ ,  $|z - i| < 1$  conformally onto the half-plane  $Im w > 0$ . If you use a composition of maps, you need only indicate the individual components.