MA 523, Qualifying Exam — Professor Donnelly
August 2016

1. Show that there are no solutions of the linear equation $u_x + u_y = u$ which pass through the straight line $x = t$, $y = t$, $u = 1$.

2. Solve the initial boundary value problem $u_t = ku_{xx}$, $u(0,t) = 0$, $u(\pi,t) = 0$, $u(x,0) = \sin x$ when $0 \leq x \leq \pi$, $t > 0$, and k is a constant.

3. Show that the problem $y_{tt} = a^2 y_{xx} + \phi(x,t)$, $0 < x < c$, $t > 0$, with boundary conditions $y(0,t) = p(t)$, $y(c,t) = q(t)$, $t \geq 0$ and initial conditions $y(x,0) = f(x)$, $y_t(x,0) = g(x)$ has at most one solution $y(x,t)$ which is twice continuously differentiable.

4. Show that $\dfrac{x^2 - y^2}{(x^2+y^2)^2}$ is harmonic, except at the origin.

5. For a function $h(x,y)$ which is harmonic in the domain $|x| < 1,\ |y| < 1$, one on the side $x = -1$ and zero on the remainder of the boundary, what is the value of h at the origin? Assume that $h(x,y)$ is continuous on the closed square, $|x| \leq 1$, $|y| \leq 1$.