Math 519 – B. Davis
Qualifying Examination
January, 2012

(Note: the geometric p distribution puts probability $(1-p)^k p$ on k for k a positive integer. For the exponential λ distribution, λ stands for the rate, not the expectation.)

Your answers to all the problems (except of course 3) should all be in a simple form.

(20) 1. i) Find the expected square of the distance from the center of a solid sphere of radius 1 to a point chosen at random from the interior of the sphere.
ii) Two points are selected at random from the interior of a sphere of radius 1. Find the expected square of the distance between these points.

(20) 2. The random variables $X, Z,$ and W are independent. X is geometric $1/3$, while Z is exponential 2 and W is exponential 3. Find $P(X < Z < W)$.

(20) 3. Prove that if X has a normal distribution and θ is a number then the variance of $\max(X, \theta)$ is smaller than the variance of X.

(20) 4. 10 points are chosen at random on a circle of radius one. A point is lonely if no other point is within a circle distance of $2\pi/100$ of it. (The circle distance between two points on the circle is the length of the smallest are of the circle connecting them.) Find the expected number of lonely points and the variance of the number of lonely points.

(20) 5. Let $X, Y, Z,$ and W be independent geometric p random variables, where $0 < p < 1$. Let $T_1 = X$, $T_2 = X + Y$, $T_3 = X + Y + Z$, and $T_4 = X + Y + Z + W$. Find $P(X < 30 | T_3 < 101 \leq T_4)$.