QUALIFYING EXAMINATION

January 2008

MA 519 - Prof. Sellke

1. A box contains 2 red, 3 white, and 4 black beads. A blind person randonly arranges the 9 beads on a circular loop of string. What is the probability that the beads are grouped together by color (with the red beads together in a group, the white beads together, and the black beads together)?

2. A fair, six–sided die is rolled repeatedly until all six possible results have been obtained. Let

$$X = \text{the number of rolls to see the first } 1.$$
$$Y = \text{the number of rolls to see the first } 6.$$

   So, if the successive results are 2556431, then $Y = 4$ and $X = 7$. Find $E\{X|Y = 5\}$.

3. Let's ignore leap years and assume that there are

$$60 \times 24 \times 365 = 525,600$$

   minutes in a year. Let's also assume that each person is equally likely to have been born in any of these 525,600 minutes, with different people having independent birth minutes. Thus, the birth minutes of different people are independent discrete–uniform random variables on the set $\{1, 2, 3, \dots, 525, 600\}$.

   About how many people do you need in a room to have a 50–50 chance (that is, probability $\frac{1}{2}$) of there being at least one birth minute shared by two or more people?

4. Let X and Y be independent $U[0, 1]$ random variables. Find the density of $Z = -Y/\ln(X)$.

5. Let N be geometric (p), with $P\{N = k\} = (1-p)^{k-1}p,\ k = 1, 2, \dots,\ EN = \frac{1}{p}$, $\mathrm{var}(N) = \frac{q}{p^2}$.

   Let $X_1, X_2, \dots$ be independent unit–exponential random variables, with density $f(x) = e^{-x},\ x > 0$, and $EX_n = \mathrm{var}(X_n) = 1$. The X_n's are independent of N.

   Let

$$Y = \sum_{i=1}^{N} X_i.$$

   Find the variance of Y.