**QUALIFYING EXAMINATION**

AUGUST 2006

MATH 519 - Prof. Davis

Each problem is worth 20 points.

1. A die is rolled until two different numbers appear. Let $T$ be the total number of times the die is rolled. Find $ET$ and also find the probability that exactly one three is rolled up to and including roll number $T$.

2. Toss a quarter $n$ times. Each time the quarter comes up heads toss a nickel. Let $X_n$ equal the number of heads on the quarter, and $Y_n$ be the number of heads on the nickel. Find $E(X_n + Y_n)$ and $EX_nY_n$.

3. A circular dartboard has a radius of one foot. Three different darts are thrown at the dartboard. The darts independently hit the board at locations uniformly distributed on the face of the board. Let $D_1$, $D_2$, and $D_3$ be the distances of the darts to the center of the dartboard (so $0 \leq D_i \leq 1$), and let $0 \leq D_{(1)} \leq D_{(2)} \leq D_{(3)} \leq 1$ be the corresponding order statistics. Find $E(D_{(2)})$ and the joint density of $D_{(1)}$ and $D_{(3)}$.

4. Ten men and ten women are randomly seated around a round table. Let $N$ be the number of men whose immediate neighbors are both women. Find $EN$, $P(N = 10)$, $P(N = 9)$.

5. Find a function $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that if $U_1$ and $U_2$ are independent uniform $(0, 1)$ random variables then $f(U_1, U_2)$ is uniformly distributed on the triangle $\{(x, y) : 0 \leq x \leq 2,\ 0 \leq y \leq x\}$.