QUALIFYING EXAMINATION

AUGUST 1999

MATH 519 - Prof. DasGupta

(20) 1. The number of fish that Anirban catches on any given day has a poisson distribution with mean 20. Due to the legendary softness of his heart, he sets free, on an average, 3 out of 4 fish he catches. Find the mean and the variance of the number of fish Anirban takes home on a given day.

(20) 2. A fair die is rolled and at the same time a fair coin is tossed. This is done repeatedly. Find the probability that a head occurs (strictly) before a six occurs.

(20) 3. X, Y are independent random variables with the common density $f(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$. Find the density function of $X + Y^2$.

(20) 4. Let X_n denote the distance between two points chosen independently at random from the unit cube in $\mathbb{R}^n$. Evaluate

$$\lim_{n\to\infty} \frac{E(X_n)}{\sqrt{n}}.$$

(20) 5. Let X be distributed as Uniform $[0, 1]$. What is the probability that the digit 5 does not occur in the decimal expansion of X?