QUALIFYING EXAMINATION

JANUARY 1995

MATH 519

25 points/problem

1. Let $X_1, X_2, \ldots, X_{11}$ be independent identically distributed random variables each having density $f(t) = e^{-t}$, $t > 0$, $f(t) = 0$ elsewhere. Find the densities of the maximum of the eleven X_is, of the minimum of the eleven X_is, and of the second largest of the eleven X_is.

2. Let (X, Y) be uniformly distributed with on the <u>circumference</u> of the unit square $\{(x, y) : |x| = 1 \text{ or } |y| = 1\}$. (So, for example, $P(a < X < b \text{ and } Y = 1) = (b - a)/8$ if $-1 \leq a < b \leq 1$). Find the distributions of $X + Y$ and XY. Are X and Y independent? Find the joint density of $\Theta(X, Y)$, where Θ is a random rotation of a point about $(0, 0)$ through an angle uniformly distributed on $[0, 2\pi]$.

3. Let $X_1, \ldots, X_{100}$ be independent normal $(0, 1)$ variables. Let $Y_1, \ldots, Y_{100}$ be independent variables each with distribution $P(Y_i = 1) = P(Y_i = -1) = \frac{1}{2}$. Let $Z_1, \ldots, Z_{100}$ be independent variables each with a uniform distribution on $(-1, 1)$. Suppose all the 300 Xs, Ys, Zs are independent.

   i) Is $\displaystyle\sum_{i=1}^{100} X_i$ <u>exactly</u> normal? If not, why not?

   ii) Is $\displaystyle\sum_{i=1}^{100} Y_i$ <u>exactly</u> normal? If not, why not?

   iii) Is $\displaystyle\sum_{i=1}^{100} Z_i$ <u>exactly</u> normal? If not, why not?

   iv) Estimate $P(\displaystyle\sum_{i=1}^{100} X_i Y_i Z_i > 10)$, in terms of the standard normal distribution function.

   v) Find $P(\displaystyle\prod_{i=1}^{100} X_i \prod_{i=1}^{100} Y_i \prod_{i=1}^{100} Z_i > 0)$.

4. A die is rolled until two sixes in a row come up. Find the expected number of rolls and the expected total of all the numbers rolled. Also find the distribution

of the number of sixes rolled and the distribution of the number of fives rolled.