Name:__________ **Score:**__________

Numerical Analysis Qualifying Exam, January 2018

Note:

1. **No books and no notes in this test. Calculators are allowed.**
2. **There are 4 problems in this exam.**
3. **Show necessary intermediate steps of your work. no credit otherwise.**

1. Consider the initial value problem

$$\frac{dy}{dt} = \cos(ty) + 1, \qquad y\left(\frac{\pi}{2}\right) = 1.$$

Derive a third-order Taylor-series method for computing $y\left(\frac{\pi}{2}+h\right)$, where h is the step size.

2. Determine the Gaussian quadrature formula of the form

$$\int_0^\infty e^{-x} f(x)dx \approx A_0 f(x_0) + A_1 f(x_1), \qquad x_0 \leqslant x_1. \tag{1}$$

3. Prove the relation

$$\begin{aligned} f(x) = & f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + \dots \\ & + f[x_0,x_1,\dots,x_n](x-x_0)(x-x_1)\dots(x-x_{n-1}) \\ & + f[x_0,x_1,\dots,x_n,x](x-x_0)(x-x_1)\dots(x-x_{n-1})(x-x_n), \end{aligned}$$

where x_i $(0 \leqslant i \leqslant n)$ are a set of distinct points.

4. Suppose $f(x)$ has continuous second derivative and r is a simple root of $f(x)$ and $f''(r) \neq 0$. Consider the Newton's method for computing r.
   (a) Derive the error relation for the Newton's method.
   (b) Based on the error relation, show that the Newton's method is locally convergent for computing r, and that its convergence rate is quadratic.