Score:___________

Numerical Analysis Qualifying Exam, August 2018

Note:

1. **No books and no notes in this test. Calculators are allowed.**
2. **Show intermediate steps of your work. no credit otherwise.**

1. Assume single precision in the computations for this problem.
   (a) (10 points)
   x, y and z are real numbers. The exact values of x and y are $(2^{11} + 2^{-13} + 2^{-14})$ and $(2^{-13} + 2^{-15})$, respectively. z is computed by $z = x + y$. Determine what value one will get for z on the computer.
   (b) (10 ponts)
   Determine the relative error $\left|\frac{fl(0.1)-0.1}{0.1}\right|$, where $fl(x)$ denotes the floating-point number corresponding to x.

2. (10 points)
   Suppose $f(x)$ has continuous second derivative, and r is a simple root of $f(x)$ with $f''(r) \neq 0$. A root-finding method leads to the following error relation for computing r (x_n denoting the approximation at step n)

$$e_{n+1} = \frac{f''(\xi)}{f'(\eta)} e_n e_{n-1}$$

   where e_n is the error of step n, ξ is some value between x_n and r, and η is some value between x_n and x_{n-1}. Show that this method is locally convergent for computing r, that is, the method will converge if the two initial guesses are sufficiently good.

3. (10 points)
   Prove that the iteration, $x_{n+1} = e^{-x_n} + 1$, converges starting with an arbitrary x_0 on the real axis.

4. (a) (10 points)
   Given natural cubic spline function $f(x)$ having knots $\{\ -1,\ 0,\ 2\ \}$ defined by

$$f(x) = \begin{cases} g(x), & x \in [-1, 0] \\ ax^4 + x^3 + bx^2, & x \in [0, 2] \end{cases}$$

   where a and b are constants and $g(x)$ is some function. Determine the values $f(-\frac{1}{2})$ and $f(1)$.

(b) (10 points)
Let $B_i^k(x)$ denote the B-spline function of degree k having knots t_i $(i = \ldots, -1, 0, 1, \ldots)$. Prove that

$$\sum_{i=-\infty}^{\infty} t_{i+1}t_{i+2}t_{i+3}B_i^3(x) = x^3.$$

5. (10 points)
Determine the Gaussian quadrature formula of the form

$$\int_0^{\infty} e^{-x} f(x)dx \approx A_0 f(x_0) + A_1 f(x_1), \quad x_0 \leqslant x_1.$$

6. (10 points)
Determine the coefficients A_i $(i = 0, 1, 2)$ in the third-order Adams method

$$y_{n+1} - y_n = A_0 f_{n+1} + A_1 f_n + A_2 f_{n-1}$$

for solving $\frac{dy}{dt} = f(t, y)$, where $f_n = f(t_n, y_n)$. Assume the time step size is h.