Introduction To Partial Differential Equations
Instructor: Professor Changyou Wang
Course Number: MA 52300
Credits: Three
Time: 12:00–1:15 PM TTh

Description

First order quasi-linear equations; the Cauchy-Kovalevsky theorem; characteristics, classification and canonical forms of linear equations; qualitative study of Laplace, Heat and Wave equations; methods of solution. Typically offered once per academic year.

Introduction To Abstract Algebra
Instructor: Professor Daniel Le
Course Number: MA 55300
Credits: Three
Time: 3:30–4:20 PM MWF

Description

Basic theory of groups, rings, and fields. Groups: group actions, subgroups, conjugation, and normal subgroups. Rings: ideals, modules, and principal ideal domains. Fields: field extensions, Galois theory, finite fields, and solvability.

Abstract Algebra I
Instructor: Professor Jaroslaw Wlodarczyk
Course Number: MA 55700
Credits: Three
Time: 4:30–5:45 PM TTh

Description

The course covers material from the text Introduction to Commutative Algebra by M. F. Atiyah and I. G. Macdonald. In particular, I plan to cover the following topics: rings and ideals, Zariski topology, modules, rings and modules of fractions, primary decomposition, integral dependence and valuations, chain conditions, Noetherian rings, Artin rings, discrete valuation rings and Dedekind domains, completions and dimension theory. Brief treatments of other topics illustrating connections between the algebraic notions and affine algebraic geometry will be included as time permits. The important part of the class will be solving problems from AM- book .

Text: M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra

Algetraic Number Theory
Instructor: Professor Freydoon Shahidi
Course Number: MA 58400
Credits: Three
Time: 10:30–11:20 AM MWF

Description

This course aims at building the foundation for studying number theory and many other fields, directly or indirectly influenced by it such as arithmetic and algebraic geometry, analytic number theory, automorphic forms, representations of groups over local and global fields, and even algebraic topology. It is the first in a two semester courses, with the second semester covering class field theory, one of the crowning achievements of number theory in the 20th century and still an extremely influencial subject that appears in different shapes and forms in these subjects. In fact, the Langlands program has been an effort to extend the (abelian) class field theory to the mysterious non-abelian setting.

Topics include: Norms and traces, integers in number fields, Dedekind domains, localization, field extensions, cyclotomic fields, Gauss sums, Artin symbol, quadratic reciprocity, local fields in complete details including Hilbert ramification theory, different and discriminant, Dirichlet unit theorem, finiteness of class numbers, adeles an ideles (time permiting)

References: My notes and Books in Algebraic Number Theory by C. Janusz; S. Lang; E. Artin; J. Neukirch

Harmonic Analysis And Applications
Instructor: Professor Victor Lie
Course Number: MA 59800AHA
Credits: Three
Time: 12:00–1:15 PM TTh

Description

This will discuss an array of fundamental concepts, starting with basic Calderon-Zygmund theory, Hardy-Littlewood maximal function, real (Marcinkiewicz) and complex interpolation, elements of functional analysis (open mapping, closed graph and Hahn-Banach theorems) with applications in Fourier Series, Fourier Transform, elements of restriction theory with applications in PDE (local wellposedness for some fundamental PDE and Strichartz estimates), Kakeya problem over finite fields etc. A good reference for this class would be the Fourier Series and Functional Analysis books by Stein and Shakarchi but the material stated above would only partly overlap. Other useful materials include Fourier Analysis lecture notes of Terry Tao slides prepared by me over the years and some research papers. [NOTE: There exists some flexibility in choosing supplementary topics based on the interests of the auditorium. Once we lay the foundation, after the mid of the semester, we could include some very recent research topics such as elements of decoupling theory or a discussion on classes of singular/maximal operators related to Carleson operator, (Bi)linear Hilbert transform along curves and relation to Zygmund Conjecture on the differentiability along Lipschitz vector fields.

Introduction to K-theory
Instructor: Professor Marius Dadarlat
Course Number: MA 59800AKT
Credits: Three
Time: 1:30–2:45 PM TTh

Prerequisites: Some prior exposure to elements of algebraic topology and very basic functional analysis would be helpful but not absolutely necessary.

Description

The course is meant as a friendly introduction to Topological K-theory. Topics will include:

1. Some basic properties of Banach algebras
2. Vector bundles and Swan's theorem
3. K-theory groups and Bott periodicity (in the complex case).
4. Multiplicative structure on K-theory and some classic applications
5. Characteristic classes
6. A brief discussion of the Atiyah-Singer Theorem (to the extent that time would allow for it).

Most of the material can be found in the following:

References:

[1] A. Hatcher, *Vector Bundles and K-theory*
(available online: https://pi.math.cornell.edu/ hatcher/VBKT/VB.pdf)

[2] E. Park, *Complex Topological K-Theory*, Cambridge University Press 2008

Grades: will be based on attendance, class participation and on a few homework assignments.

Introduction to Optimal Transport
Instructor: Professor Yuan Gao
Course Number: MA 59800AOT
Credits: Three
Time: 9:00–10:15 AM TTh

Description

This is a graduate-level course introducing the subject of optimal transport. The mathematical theory of optimal transport can be formulated in terms of probability measures on an underlying metric space; the problem of mapping or transporting one measure to another leads to the definition of a metric on the set of probability measures which reflects the geometry of the underlying space. This theory has connections to many topics within analysis, probability, geometry, statistics and data science. The first part of the course will develop the basic theory of optimal transport including Monge problem, the dual formulation, polar factorization, the Monge-Ampere equation, the Wasserstein metric and the Benamou-Brenier method. The rest of the course will cover application of optimal transport ideas to partial differential equations (PDE), convex analysis, stochastic algorithm and data science. The goal of the course is to introduce the fundamental mathematical ideas in the field and to give a survey of some recent applications to other related areas.

Methods of Applied Mathematics I
Instructor: Professor Isaac Harris
Course Number: MA 64200
Credits: Three
Time: 2:30–3:20 PM MWF

Description

Second order elliptic equations including maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates. Applications of linear theory to nonlinear equations.

Topics List:

- Distributions and Sobolev Spaces
- Embeddings and Norm Estimates
- Theorems for Well-posedness
- Linear and Quasi-Linear Elliptic BVPs
- Regularity of Solutions
- Applications to Inverse Problems

Reference Texts (optional):

"Partial Differential Equations in Action: From Modeling to Theory" by S. Salsa

"Variational Techniques for Elliptic Partial Differential Equations" by F. Sayas, T. Brown and M. Hassell

Modern Differential Geometry
Instructor: Professor Ben McReynolds
Course Number: MA 66100
Credits: Three
Time: 10:30–11:45 AM TTh

Introduction to Algebraic Geometry
Instructor: Professor Tong Liu
Course Number: MA 66500
Credits: Three
Time: 11:30 AM–12:20 PM MWF

Description

We shall give a basic course in Algebraic Geometry based upon Hartshorne's Algebraic Geometry: Chapter 1 and Chapter 2. That is, we will mainly discuss basic ideas and properties for algebraic varieties and schemes.

Homework: Problem solving is vital for this class. I will actively seek groups of volunteers to report their solutions in the problem sessions. There will be usually a 30 minutes problems session held during Friday's lecture. During this time the (previously selected) volunteers are going to present their solutions.

Commutative Algebra: It is very useful to have basic working knowledge of commutative algebra, at the level of Introduction to Commutative Algebra by Atiyah and Macdonald, before plunging into Hartshorne's book.

Exams: No exam.

Topics in Commutative Algebra: Complete Intersections
Instructor: Professor Linquan Ma
Course Number: MA 69000ACI
Credits: Three
Time: 1:30–2:45 PM TTh

Description

We will discuss complete intersection rings and the module theory over them, following celebrated works of Avramov and of Avramov-Buchweitz. To achieve this, we will set up some foundations on differential graded algebra (dga). If time permitting, we will also discuss recent (remarkable) works of Briggs and of Briggs-Iyengar characterizing complete intersections via conormal modules and cotangent complexes.

Randomized Numerical Linear Algebra
Instructor: Professor Jianlin Xia
Course Number: MA 69200ARN
Credits: Three
Time: 3:00–4:15 PM TTh

Description

This course will cover randomized methods and theories for numerical linear algebra. It will include the background, essential tools, and latest developments of randomized linear algebra. The main topics include:

- Motivations for randomization in matrix computations
- Statistical tools
- Random matrices
- Probabilistic estimators (norm, trace, etc.)
- Randomized low-rank approximations (sketching, Nystrom, SVD, pivoting, etc.)
- Randomized solvers and preconditioners for linear systems
- Stochastic optimization
- Randomized eigenvalue solvers
- Connections to structured matrices, PDE solutions, data analysis, and machine learning

Introduction to Quantum invariants and Volume Conjectures
Instructor: Professor Xingshan Cui
Course Number: MA 69700
Credits: Three
Time: 10:30–11:20 AM MWF

Description

This course is an introduction to the concept of quantum invariants and, more broadly, topological quantum field theories, which have been developed over the past three decades into a subject called quantum topology. The starting point for the subject is the discovery of the Jones polynomial of a knot and the formulation of it as a quantum field theory in physics in the 1980s. We will focus on the mathematical side of the theory and hence no physics background is required. Quantum invariants usually refer to invariants of knots/manifolds that are constructed in a ‘similar style? as the Jones polynomial, and are named so to contrast with classical invariants such as homology/homotopy groups.

A central problem in this area is the so-called volume conjecture that relates the colored Jones polynomial of a hyperbolic knot to the hyperbolic volume of the knot. There are a few versions of volume conjectures using different quantum invariants. Special cases of the conjectures have been verified but a full solution remains open.

In the first half of the course, we will define the colored Jones polynomial and generalize it to the Reshetikhin-Turaev invariants of 3-manifolds. Along the way, we will touch some topics on knot theory, skein theory, and tensor categories. In the second half, we will review some basics in hyperbolic geometry in order to formulate the volume conjecture. Towards the end, we will sketch the proof for some special cases of the conjecture. The course is aimed to be self-contained and some basic backgrounds in knot/3-manifold theory, category theory, and differential geometry will be useful, but not strictly required.

Textbook: There is not a textbook. Some relevant references include,

1). V. Turaev, Quantum Invariants of Knots and 3-Manifolds

2). Z. Wang, Topological Quantum Computation

3). W. Thurston, The Geometry and Topology of Three-Manifolds

4). H. Murakami, An Introduction to the Volume Conjecture