Abstract Algebra I
Instructor: Professor Daniel Le
Course Number: MA 55700
Credits: Three
Time: 10:30–11:45 AM TTh

Description

Commutative algebra grew out of concurrent developments in number theory, the function theory of Riemann surfaces, invariant theory, and algebraic geometry. This course will cover basic notions in commutative algebra, highlighting some of the applications that gave the theory its impetus. These include rings, ideals, modules, localization, primary decomposition, and Noetherianity.

References:

- Atiyah and McDonald, Introduction to Commutative Algebra
- Matsumura, Commutative Ring Theory
- Eisenbud, Commutative Algebra with a View Toward Algebraic Geometry

Introduction to Differential Geometry & Topology
Instructor: Professor Kiril Datchev
Course Number: MA 56200
Credits: Three
Time: 9:30–10:20 AM MWF

Description

This course is an introduction to smooth manifolds and to the differential and integral calculus of functions, vectors, and tensors. It serves as a foundation for more advanced work in geometry, topology, and differential equations. The material also appears in more advanced work in other disciplines, including electromagnetism, fluid mechanics, and relativity.

Topics will include a review of analysis in several variables, differentiable manifolds and submanifolds, vector fields, tensors and tensor fields, differential forms, integration, Stokes' theorem. Time permitting, we will also introduce Riemannian manifolds, geodesics, and curvature.

Text books: An Introduction to Differentiable Manifolds and Riemannian Geometry, Second Edition, by William M. Boothby

Topics in Harmonic/Fourier Analysis
Instructor: Professor Rodrigo Bañuelos
Course Number: MA 59800ACZ
Credits: Three
Time: 2:30–3:20 PM MWF

Description

This course will cover some of the basic tools in harmonic and Fourier analysis that are extremely useful in other areas such as PDE's, stochastic analysis and complex analysis. Specific topics covered in the course include:

(1) Geometric theorems of Vitali, Wiener, and Whitney & applications to maximal function.
(2) Convolutions, approximations to the identity and applications to boundary value problems in $\mathbb{R}^{nd}$ with L^p-data.
(3) The Marcinkiewicz & Riesz–Thorin interpolation theorems.
(4) Fourier transform.
(5) The Calderón-Zygmund singular integral theory for convolutions kernels satisfying Hörmander's conditions. The classical examples of the Hilbert and Riesz transforms will be discussed. Applications to Littlewood-Paley theory & the Hörmander-Mikhlin multiplier theorem.
(6) Hardy-Littlewood-Sobolev inequalities for fractional integration and powers of the Laplacian and the inequalities of Nash and Sobolev. Some of these topics will be presented in the general setting of heat semigroups and Dirichlet forms under the assumption of "Varopoulos finite dimension." They include those arising from various stochastic processes such as Brownian motion, elliptic diffusions, symmetric stable process and more general Lévy processes.

Prerequisites: Math 544. Depending on need, some topics may be reviewed.

Text books: No text book is required. The course follows my book "Lecture in Analysis" which you can download https://www.math.purdue.edu/ banuelos/Lectures/BanuelosAnalysisNotes.pdf Recommended: (1), L. Grafakos *"Modern Fourier Analysis,"* (2) E. M. Stein *"Singular Integrals and Differentiability Properties of Functions."*

Required work: From time to time some problems may be assigned to clarify topics covered in class. Interested students will have the opportunity to read and present a paper on topics related to the course and as relevant as possible to their interests and background.

Deep Reinforcement Learning
Instructor: Professor Haizhao Yang
Course Number: MA 598ADL
Credits: Three
Time: 10:30–11:45 AM TTh

Description

This course provide a basic introduction to deep reinforcement learning, applications of deep reinforcement learning in mathematical science and scientific computing, and theoretical analysis of deep reinforcement learning.

Algebraic Geometry – Introduction to Schemes
Instructor: Professor Jaroslaw Wlodarczyk
Course Number: MA 59800AIS
Credits: Three
Time: 4:30–5:45 PM TTh

Description

We shall give a course in Algebraic Geometry based upon Hartshorne's *Algebraic Geometry*: Chapters 2 and Vakil's *Rising Sea*. The important part of the class will be solving problems from

Hartshorne's book and others. The emphasis of this course is on the proofs and algebraic tools used in Algebraic Geometry.

We plan to cover Chapter 2 of Hartshorn's book (most of the sections) including most of the exercises. Selected problems from Hartshorne's book will be assigned as homeworks.

The tentative list of the topics includes but is not limited to:

- Sheaves
- Properties of schemes
- Separated and proper morphisms
- Regularity
- Normal varieties, Normalization
- Coherent and Quasicoherent modules
- Projective morphisms
- Differentials, and formal smoothness
- Divisors, Cartier divisors, invertible sheaves
- Formal schemes (if time permits)

Texbook and course notes:

The textbooks is *Algebraic geometry* (AG) by Hartshorne.

Vakil's notes (V)

Additional reading

- Basic Algebraic Geometry (Part 1) (BAG) by Igor Shafarevich
- *Introduction to commutative algebra* (AM) by Atiyah and Macdonald
- *Commutative algebra with a view towards algebraic geometry by Eisenbud (E)*

Homework:

There is no grader for this course. I encourage you to solve as many homework problems as you can. The solutions of homework problems will be presented by volunteers in the class or by me.

Topological Data Analysis

Instructor: Professor Ralph Kaufmann

Course Number: MA 59800ATDA

Credits: Three

Time: 10:30–11:20 AM MWF

Description

Topological data analysis, as the name says, is right the junction of pure mathematics and data applications. It is a recent tool designed to capture structural features of data sets and has found wide applications in diverse fields.

This course will be an introduction to the subject with the intent to provide both a working knowledge and a deeper understanding of the theory that provide the flexibility needed to solve real world problems as well as to further the theoretical toolkit.

The course is aimed at non-experts and is intended to be accessible to a wide audience. All mathematical notions will be developed from scratch, running the gamut from theory to application.

In the first part, we will introduce the main notions of persistent homology and bar codes. For this we will review the essential features of the homology that is used and the computational aspects. We will give precise definitions of the workings of persistence and introduce persistence diagrams. These concepts are put into a broader context compared with notions from other areas. As concrete examples we will consider various discrete and graphical setups relating to nerve constructions, such as Cech, Vietoris-Rips, Voronoi, Delaunay, and Alpha complexes. We will then discuss various metrics and measures of stability, such as the bottleneck distance.

In the second part of the course, we will discuss further topics which may include Reeb graphs, visualization using the Mapper algorithm, principal component analysis, and various clustering algorithms. This will also depend on the interests of the audience.

As a third component, it is planned to briefly touch upon programming using the jupyter interface.

The plan is to have an end project, which could either be a presentation of a paper in the subject or a small programming project.

Commutative Algebra
Instructor: Professor Bernd Ulrich
Course Number: MA 65000
Credits: Three
Time: 4:30–5:20 PM MWF

Description

This is an intermediate course in commutative algebra. The course is a continuation of MA 55700/55800, but should be accessible to any students with basic knowledge in commutative algebra and homological algebra (localization, Noetherian and Artinian modules, associated primes, dimension theory, Ext and Tor).

The topics of this semester will include: Regular sequences, depth and Cohen–Macaulayness, structure of finite free resolutions, regular rings and normal rings, canonical modules and Gorenstein rings, local cohomology and local duality.

Prerequisites: Basic knowledge of commutative algebra.

Texts: No specific text will be used, but possible references are:

- H. Matsumura, Commutative ring theory, Cambridge University Press
- W. Bruns and J. Herzog, Cohen–Macaulay rings, Cambridge University Press
- D. Eisenbud, Commutative algebra with a view toward algebraic geometry, Springer.

Grobner Bases Commutative Algebra
Instructor: Professor Giulio Caviglia
Course Number: MA 69000B
Credits: Three
Time: 3:30–4:20 PM MWF

Description

The course will focus on the theory of Grobner bases and its applications in Commutative Algebra. I will discuss both theoretical and computational aspects. Three central topics will be covered:

- Graded free resolutions and their homological invariants.
- Generic initial ideals.
- Determinantal ideals.

Structured and Randomized Matrix Computations
Instructor: Professor Jianlin Xia
Course Number: MA 69200M
Credits: Three
Time: 1:30–2:45 PM TTh

Description

In this course, we will cover both topics of structured matrices and randomized linear algebra. We will discuss fundamental methods and theories as well as advanced numerical techniques. The structured matrix part will focus on rank structures and relevant fast solvers. The randomized linear algebra part will include various statistical and randomized techniques for performing large matrix computations. In particular, we will show how structured matrix methods and randomization can be used together to solve numerical problems such as challenging discretized PDEs and large eigenvalue problems. Selected applications to data science, imaging, and engineering will also be discussed.

Complex Algebraic Geometry and Hodge Theory
Instructor: Professor Donu Arapura
Course Number: MA 69600CAG
Credits: Three
Time: 12:00–1:15 PM TTh

Description

Algebraic geometry is roughly the study of the set of solutions of polynomial equations over a field, or more generally over a commutative ring. In this generality, the techniques are purely algebraic, but over the field of complex numbers, these can be supplemented by the methods from algebraic topology, complex analysis, and differential geometry. To appreciate why this is useful, consider the definition of the genus of a curve. The most intuitive definition is to draw a picture, and the count the number of "holes", or more rigorously as one half the first Betti number. Why this coincides with the more algebraic definition is nontrivial. At the very least, it involves the Hodge and de Rham theorems, and these are the sorts of things we will discuss at the beginning. We will also go through the Lefschetz theorems (weak and "hard" versions) in some detail. Beyond that, I don't have a fixed agenda. We will see how it goes.

I don't have a specific list of prerequisites, but if you're in doubt, then just ask me.

While I won't really follow any book, most of what I talk about is contained in the union of the following references:

1. Griffiths, Harris, Principles of algebraic geometry
2. Peters, Steenbrink, Mixed Hodge structures
3. Voisin, Hodge theory and Complex algebraic geometry I, II

Introduction to Hochschild and Cyclic Homology
Instructor: Professor Manuel Rivera
Course Number: MA 69700H
Credits: Three
Time: 1:30–2:20 PM MWF

Description

The theory Hochschild homology and cohomology is a basic tool to study algebras and their deformations.

In non-commutative geometry the Hochschild and cyclic homology constructions provide a generalization of the classical integral and differential calculus of the commutative algebra of functions on a geometric space applicable to non-commutative algebras. The applications of the theory from the non-commutative geometry perspective include generalizations of index theorems, deformation quantization of Poisson structures.

In algebraic topology, Hochschild and cyclic homology arise in the study of loop spaces and more generally spaces with a circle action and they are related to iterated integrals. The algebraic topology perspective on Hochschild and cyclic homology has evolved in different directions. A particular direction that will be of interest in this course is string topology which introduces intricate structures on the homology of loop space.

The theory of Hochschild and cyclic homology has also illuminated many interactions between different fields of mathematics such as algebra, topology, geometry, and analysis. For example, cyclic homology has also been used as an approximation to algebraic K–theory.

In this course we will carefully develop the foundations and basics of Hochschild and cyclic homology of associative algebras assuming minimal requirements: we will assume basic homological algebra as discussed in an algebraic topology course on homology and cohomology such as 572. Knowledge of algebraic geometry will be helpful but not required. The course will be roughly divided into three parts:

1) The basics of Hochschild and cyclic homology motivated by non-commutative geometry
2) The relationship of Hochschild and cyclic homology to loop spaces and cyclic spaces
3) Algebraic operations on on Hochschild and cyclic complexes